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Drinfel'd realization of quantum affine superalgebra $U_q(\widehat{gl(1|1)})$

Jin-fang Cai†, Shi-kun Wang‡§, Ke Wu† and Wei-zhong Zhao†

† Institute of Theoretical Physics, Academia Sinica, Beijing, 100080, People's Republic of China
‡ CCAST (World Laboratory), PO Box 3730, Beijing, 100080, People's Republic of China
§ Institute of Applied Mathematics, Academia Sinica, Beijing, 100080, People's Republic of China

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Abstract. Using the super Reshetikhin–Semenov-Tian-Shansky method and Gauss decomposition, we obtain Drinfel'd's currents realization of the quantum affine superalgebra $U_q(\widehat{gl(1|1)})$. The Hopf structure for Drinfel'd's currents is also given.

There are three methods for constructing quantum algebras including quantum affine algebras and Yangians. One was given by Drinfel'd [1] and Jimbo [3, 4] independently to the quantum universal enveloping algebra $U_q(g)$ of any simple Lie algebras g. Later Drinfel'd [2] gave his second definition (or realization) of quantum affine algebras $U_q(\widehat{g})$ and Yangians. From views of the quantum inverse scattering method, Faddeev, Reshetikhin and Takhtajan (FRT) [5] found another realization of $U_q(g)$ by means of a solution of the Yang–Baxter equation (YBE), then Reshetikhin and Semenov-Tian-Shansky (RS) [6] used the exact affine analogue of the FRT method to obtain the third realization of quantum affine algebra $U_q(\widehat{g})$ with centre extension. The explicit isomorphism between the realizations of quantum affine algebras $U_q(\widehat{g})$ given by Drinfel'd [2] and RS [6] was established by Ding and Frenkel [7] using Gauss decomposition.

It should be pointed out that the authors of [8, 9] obtained all six-vertex and eight-vertex solutions of the YBE with spectral and coloured parameters and classified them into Baxter type and free-fermion type. [10] gives a solution of the YBE without a spectral parameter, which can be obtained if the spectral parameter is zero in a six-vertex solution of free-fermion type, and discusses a peculiar quantum algebra associated with the solution. In addition, using the RS method a quantum affine algebra was also discussed associated with a free-fermion-type solution of the YBE with spectral parameter [13]. However, the classical limit of both quantum algebras is not a Lie superalgebra or a super affine algebra although some of its relations (such as $X^2 = Y^2 = 0$) look like fermionic relations. An important concept given by Liao and Song [11] shows that, if we want to get a quantum superalgebra from the non-standard solution of the YBE, we must use the graded calculation for the YBE and RLL relations etc at the very beginning, i.e. a super version of the FRT method. Recently, some attention has been paid to the construction of the quantum affine superalgebras [12, 13]. In this paper, we use the super RS method to construct a quantum affine superalgebra and Gauss decomposition (Ding–Frenkel map) to obtain its Drinfel'd currents realization. The Hopf structure for these currents is computed straightforwardly

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from the original one defined in the RS method, however, it differs from that in [17]. We will focus on the simplest one, $U_q(\widehat{gl(1|1)})$, and this method can be easily extended to the general case $U_q(\widehat{gl(m|n)})$.

We denote V as a $\mathbb{Z}_2$-graded two-dimensional vector space (graded auxiliary space), and set the first basis of V as even (bosonic) and the second as odd (fermionic). In this case, the multiplication rule of the tensor product is $(A \otimes B)(C \otimes D) = (-1)^{P(B)P(C)} AC \otimes BD$, where $P(A) = 0, 1$ when A is bosonic or fermionic. Then the graded (super) YBE takes the following form [11]

$$\eta_{12} R_{12}(z/w) \eta_{13} R_{13}(z) \eta_{23} R_{23}(w) = \eta_{23} R_{23}(w) \eta_{13} R_{13}(z) \eta_{12} R_{12}(z/w) \tag{1}$$

where $R(z) \in \mathrm{End}(V \otimes V)$. The R-matrix must obey the weight conservation condition $R_{ij,kl} \neq 0$ only when $P(i) + P(j) = P(k) + P(l)$, where $P(1) = 0$ and $P(2) = 1$. The components of the factor η are $\eta_{ik,jl} = (-1)^{P(i)P(k)} \delta_{ij} \delta_{lk}$. It is obvious that $\eta R(z)$ satisfies the ordinary YBE when $R(z)$ is a solution of the super YBE. The super YBE can also be written in components as

$$R^{ij}_{ab}(z/w) R^{ak}_{pc}(z) R^{bc}_{qr}(w) (-1)^{(P(a)-P(p))P(b)} = (-1)^{P(e)(P(f)-P(r))} R^{jk}_{ef}(w) R^{if}_{dr}(z) R^{de}_{pq}(z/w). \tag{2}$$

It can be verified that the following R-matrix [14] satisfies the graded YBE (1)

$$R_{12}(z) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{z-1}{zq - q^{-1}} & \frac{z(q-q^{-1})}{zq-q^{-1}} & 0 \\ 0 & \frac{(q-q^{-1})}{zq-q^{-1}} & \frac{z-1}{zq-q^{-1}} & 0 \\ 0 & 0 & 0 & -\frac{q - zq^{-1}}{zq - q^{-1}} \end{pmatrix}. \tag{3}$$

This solution is of free-fermion type and also satisfies the unitary condition: $R_{12}(z) R_{21}(z^{-1}) = \mathbf{1}$. When $z = 0$ and q is replaced by q^{-1}, the $\eta R(z)$ term degenerates to the non-standard solution of the YBE which has been used in studying the quantum superalgebra $U_q(gl(1|1))$ [11]. The solution of $\eta R(z)$ can also be obtained from the non-standard solution through the Baxterization procedure [15].

From the above solution of the graded YBE, we can define the quantum affine superalgebra $U_q(\widehat{gl(1|1)})$ with a central extension employing the super RS method or the affine version of that in [11]. $U_q(\widehat{gl(1|1)})$ is an associative algebra with generators $\{l^k_{ij} | 1 \leqslant i, j \leqslant 2, k \in \mathbb{Z}\}$ and centre c, which subject to the following multiplication relations

$$R_{12}(z/w) L^{\pm}_1(z) \eta L^{\pm}_2(w) \eta = \eta L^{\pm}_2(w) \eta L^{\pm}_1(z) R_{12}(z/w) \tag{4}$$

$$R_{12}(z_-/w_+) L^{+}_1(z) \eta L^{-}_2(w) \eta = \eta L^{-}_2(w) \eta L^{+}_1(z) R_{12}(z_+/w_-) \tag{5}$$

where $z_\pm = zq^{\pm c/2}$. We have used standard notation: $L^{\pm}_1(z) = L^{\pm}(z) \otimes \mathbf{1}$, $L^{\pm}_2(z) = \mathbf{1} \otimes L^{\pm}(z)$ and $L^{\pm}(z) = (l^{\pm}_{ij}(z))_{i,j=1,2}$, $l^{\pm}_{ij}(z)$ are generating functions (or currents) of l^k_{ij}: $l^{\pm}_{ij}(z) = \sum_{k=0}^{\infty} l^{\pm k}_{ij} z^{\pm k}$.

This algebra admits the following co-product, co-unit and antipole structure

$$\triangle(l^{\pm}_{ij}(z)) = \sum_{k=1}^{2} l^{\pm}_{kj}(zq^{\pm c_2/2}) \otimes l^{\pm}_{ik}(zq^{\mp c_1/2}) (-1)^{(k+i)(k+j)} \tag{6}$$

$$\epsilon(l^{\pm}_{ij}(z)) = \delta_{ij} \qquad S({}^{st}L^{\pm}(z)) = [{}^{st}L^{\pm}(z)]^{-1} \tag{7}$$

$$\triangle(c) = c \otimes \mathbf{1} + \mathbf{1} \otimes c \tag{8}$$

$$\epsilon(c) = 0 \qquad S(c) = -c \tag{9}$$

where $c_1 = c \otimes \mathbf{1}$, $c_2 = \mathbf{1} \otimes c$ and $[{}^{st}L^{\pm}(z)]_{ij} = (-1)^{i+j} l^{\pm}_{ji}(z)$. It is easy to verify that the above co-product, co-unit and antipole structure are compatible with the associative multiplication defined by (4) and (5), i.e. all l^k_{ij} and c satisfy

$$\triangle(ab) = \triangle(a)\triangle(b) \tag{10}$$

$$m(S \otimes \mathrm{id})\triangle(a) = m(\mathrm{id} \otimes S)\triangle(a) = \epsilon(a) \cdot \mathbf{1} \tag{11}$$

$$(\epsilon \otimes \mathrm{id})\triangle = (\mathrm{id} \otimes \epsilon)\triangle = \mathrm{id} \tag{12}$$

$$S(ab) = S(b)S(a) \qquad \epsilon(ab) = \epsilon(a)\epsilon(b). \tag{13}$$

We next give the Drinfel'd currents realization of the quantum affine superalgebra. As was done in Ding and Frenkel [7], $L^{\pm}(z)$ have the following unique Gauss decompositions

$$\begin{aligned} L^{\pm}(z) &= \begin{pmatrix} 1 & 0 \\ f^{\pm}(z) & 1 \end{pmatrix} \begin{pmatrix} k_1^{\pm}(z) & 0 \\ 0 & k_2^{\pm}(z) \end{pmatrix} \begin{pmatrix} 1 & e^{\pm}(z) \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} k_1^{\pm}(z) & k_1^{\pm}(z)e^{\pm}(z) \\ f^{\pm}(z)k_1^{\pm}(z) & k_2^{\pm}(z) + f^{\pm}(z)k_1^{\pm}(z)e^{\pm}(z) \end{pmatrix} \end{aligned} \tag{14}$$

where $e^{\pm}(z)$, $f^{\pm}(z)$ and $k_i^{\pm}(z)$ $(i = 1, 2)$ are generating functions of $U_q(\widehat{gl(1|1)})$ and $k_i^{\pm}(z)$ $(i = 1, 2)$ are invertible. The inversions of $L^{\pm}(z)$ can be written as

$$L^{\pm}(z)^{-1} = \begin{pmatrix} k_1^{\pm}(z)^{-1} + e^{\pm}(z)k_2^{\pm}(z)^{-1}f^{\pm}(z) & -e^{\pm}(z)k_2^{\pm}(z)^{-1} \\ -k_2^{\pm}(z)^{-1}f^{\pm}(z) & k_2^{\pm}(z)^{-1} \end{pmatrix}. \tag{15}$$

We set

$$X^{+}(z) = e^{+}(z_-) - e^{-}(z_+) \qquad X^{-}(z) = f^{+}(z_+) - f^{-}(z_-). \tag{16}$$

To calculate the (anti-) commutation relations of $X^{\pm}(z)$ and $k_i^{\pm}(z)$ $(i = 1, 2)$, we must make use of the inversions of $L^{\pm}(z)$, the unitary property of the R-matrix and the following equivalent forms of (4) and (5)

$$L_1^{\pm}(z)^{-1}\eta L_2^{\pm}(w)^{-1}\eta R_{12}(z/w) = R_{12}(z/w)\eta L_2^{\pm}(w)^{-1}\eta L_1^{\pm}(z)^{-1} \tag{17}$$

$$L_1^{+}(z)^{-1}\eta L_2^{-}(w)^{-1}\eta R_{12}(z_-/w_+) = R_{12}(z_+/w_-)\eta L_2^{-}(w)^{-1}\eta L_1^{+}(z)^{-1} \tag{18}$$

$$L_1^{\pm}(z)R_{12}(z/w)\eta L_2^{\pm}(w)^{-1}\eta = \eta L_2^{\pm}(w)^{-1}\eta R_{12}(z/w)L_1^{\pm}(z) \tag{19}$$

$$L_1^{+}(z)R_{12}(z_+/w_-)\eta L_2^{-}(w)^{-1}\eta = \eta L_2^{-}(w)^{-1}\eta R_{12}(z_-/w_+)L_1^{+}(z) \tag{20}$$

$$L_1^{\pm}(z)^{-1}R_{21}(w/z)\eta L_2^{\pm}(w)\eta = \eta L_2^{\pm}(w)\eta R_{21}(w/z)L_1^{\pm}(z)^{-1} \tag{21}$$

$$L_1^{+}(z)^{-1}R_{21}(w_+/z_-)\eta L_2^{-}(w)\eta = \eta L_2^{-}(w)\eta R_{21}(w_-/z_+)L_1^{+}(z)^{-1}. \tag{22}$$

The calculation process is similar to that in [7] for quantum affine algebras. From (4) (5) and (17)—(22), we can calculate all relations between $k_i^{\pm}(z)$ $(i = 1, 2)$ and $X^{\pm}(w)$ as follows

$$[k_1^{\pm}(z), k_1^{\pm}(w)] = [k_1^{+}(z), k_1^{-}(w)] = 0 \tag{23}$$

$$[k_1^{\pm}(z), k_2^{\pm}(w)] = [k_2^{\pm}(z), k_2^{\pm}(w)] = 0 \tag{24}$$

$$\frac{w_+q - z_-q^{-1}}{z_-q - w_+q^{-1}} k_2^{+}(z)^{-1}k_2^{-}(w)^{-1} = \frac{w_-q - z_+q^{-1}}{z_+q - w_-q^{-1}} k_2^{-}(w)^{-1}k_2^{-}(z)^{-1} \tag{25}$$

$$\frac{z_\pm - w_\mp}{z_\pm q - w_\mp q^{-1}} k_1^\pm(z) k_2^\mp(w)^{-1} = \frac{z_\mp - w_\pm}{z_\mp q - w_\pm q^{-1}} k_2^\mp(w)^{-1} k_1^\pm(z) \tag{26}$$

$$k_i^\pm(z)^{-1} X^+(w) k_i^\pm(z) = \frac{z_\pm q - wq^{-1}}{z_\pm - w} X^+(w) \qquad (i = 1, 2) \tag{27}$$

$$k_i^\pm(z) X^-(w) k_i^\pm(z)^{-1} = \frac{z_\mp q - wq^{-1}}{z_\mp - w} X^-(w) \qquad (i = 1, 2) \tag{28}$$

$$\{X^+(z), X^+(w)\} = \{X^-(z), X^-(w)\} = 0 \tag{29}$$

$$\{X^+(z), X^-(w)\} = (q - q^{-1}) \left[\delta\left(\frac{w_-}{z_+}\right) k_1^-(z_+)^{-1} k_2^-(z_+) - \delta\left(\frac{z_-}{w_+}\right) k_1^+(w_+)^{-1} k_2^+(w_+) \right] \tag{30}$$

where $\delta(z) = \sum_{k\in\mathbb{Z}} z^k$. It is very clear that $X^\pm(z)$ (or $e^\pm(z)$ and $f^\pm(z)$) are of fermionic type for their anti-commutation relations and $k_i^\pm(z)$ $(i = 1, 2)$ are bosonic-type elements in $U_q(\widehat{gl(1|1)})$ as expected. This result differs from that in [13]: the relation between $X^+(z)$ and $X^-(w)$ (30) is also anti-commutator which is a requirement of the superalgebra.

Introducing a transformation for the generating functions $X^\pm(z)$ and $k_i^\pm(z)$ $(i = 1, 2)$ to obtain the currents corresponding to generators of $gl(1|1)$

$$E(z) = X^+(zq) \qquad F(z) = X^-(zq) \tag{31}$$

$$K^\pm(z) = k_1^\pm(zq)^{-1} k_2^\pm(zq) \qquad H^\pm(z) = k_2^\pm(zq) k_1^\pm(zq^{-1}) \tag{32}$$

then the (anti-) commutation relations for $E(z)$, $F(z)$, $K^\pm(z)$ and $H^\pm(z)$ are

$$[K^\pm(z), K^\pm(w)] = [H^\pm(z), H^\pm(w)] = 0 \tag{33}$$

$$[K^+(z), K^-(w)] = [K^\pm(z), H^\pm(w)] = 0 \tag{34}$$

$$H^+(z) H^-(w) = \left(\frac{(z_- q - w_+ q^{-1})(z_+ q^{-1} - w_- q)}{(z_+ q - w_- q^{-1})(z_- q^{-1} - w_+ q)} \right)^2 H^-(w) H^+(z) \tag{35}$$

$$K^\pm(z) H^\mp(w) = \frac{(w_\mp q - z_\pm q^{-1})(z_\mp q - w_\pm q^{-1})}{(w_\pm q - z_\mp q^{-1})(z_\pm q - w_\mp q^{-1})} H^\mp(w) K^\pm(z) \tag{36}$$

$$[K^\pm(z), E(w)] = [K^\pm(z), F(w)] = 0 \tag{37}$$

$$E(w) H^\pm(z) = \frac{z_\pm q - wq^{-1}}{z_\pm q^{-1} - wq} H^\pm(z) E(w) \tag{38}$$

$$F(w) H^\pm(z) = \frac{z_\mp q^{-1} - wq}{z_\mp q - wq^{-1}} H^\pm(z) F(w) \tag{39}$$

$$\{E(z), E(w)\} = \{F(z), F(w)\} = 0 \tag{40}$$

$$\{E(z), F(w)\} = (q - q^{-1}) \left[\delta\left(\frac{w_-}{z_+}\right) K^-(z_+) - \delta\left(\frac{z_-}{w_+}\right) K^+(w_+) \right]. \tag{41}$$

The above relations are Drinfel'd's currents realization of quantum affine superalgebra $U_q(\widehat{gl(1|1)})$. Setting $e^\pm(z_\mp q) = E^\pm(z)$ and $f^\pm(z_\pm q) = F^\pm(z)$, then $E(z) = E^+(z) - E^-(z)$ and $F(z) = F^+(z) - F^-(z)$. The co-product of the generating functions $E^\pm(z)$, $F^\pm(z)$, $K^\pm(z)$ and $H^\pm(z)$ can be calculated directly from (6)

$$\begin{aligned} \triangle(E^\pm(z)) &= \triangle(l_{11}^\pm(z_\mp q)^{-1} l_{12}^\pm(z_\mp q)) \\ &= [l_{11}^\pm(zq^{1\mp c_1/2}) \otimes l_{11}^\pm(zq^{1\mp c_1 \mp c_2/2}) - l_{21}^\pm(zq^{1\mp c_1/2}) \otimes l_{12}^\pm(zq^{1\mp c_1\mp c_2/2})]^{-1} \\ &\quad \times [l_{12}^\pm(zq^{1\mp c_1/2}) \otimes l_{11}^\pm(zq^{1\mp c_1\mp c_2/2}) + l_{22}^\pm(zq^{1\mp c_1/2}) \otimes l_{12}^\pm(zq^{1\mp c_1\mp c_2/2})] \end{aligned}$$

$$
\begin{aligned}
&= [\mathbf{1}\otimes\mathbf{1} - l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} l_{21}^{\pm}(zq^{1\mp c_1/2}) \otimes l_{11}^{\pm}(zq^{1\mp c_1 \mp c_2/2})^{-1} l_{12}^{\pm}(zq^{1\mp c_1\mp c_2/2})]^{-1} \\
&\quad \times [l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} l_{21}^{\pm}(zq^{1\mp c_1/2}) \otimes \mathbf{1} + l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} l_{22}^{\pm}(zq^{1\mp c_1/2}) \\
&\quad \otimes l_{11}^{\pm}(zq^{1\mp c_1\mp c_2/2})^{-1} l_{12}^{\pm}(zq^{1\mp c_1\mp c_2/2})] \\
&= [\mathbf{1}\otimes\mathbf{1} + l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} l_{21}^{\pm}(zq^{1\mp c_1/2}) \otimes E^{\pm}(zq^{\mp c_1})] \\
&\quad \times [E^{\pm}(z)\otimes\mathbf{1} + l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} l_{22}^{\pm}(zq^{1\mp c_1/2}) \otimes E^{\pm}(zq^{\mp c_1})] \\
&= E^{\pm}(z)\otimes\mathbf{1} + K^{\pm}(zq^{\mp c_1/2}) \otimes E^{\pm}(zq^{\mp c_1}) \qquad (42)
\end{aligned}
$$

$$
\begin{aligned}
\triangle(F^{\pm}(z)) &= \triangle(l_{21}^{\pm}(z_{\pm}q) l_{11}^{\pm}(z_{\pm}q)^{-1}) \\
&= \mathbf{1}\otimes F^{\pm}(z) + F^{\pm}(zq^{\pm c_2}) \otimes K^{\pm}(zq^{\pm c_2/2}) \qquad (43)
\end{aligned}
$$

$$
\begin{aligned}
\triangle(K^{\pm}(z)) &= \triangle(k_1^{\pm}(zq)^{-1} k_2^{\pm}(zq)) \\
&= \triangle(l_{11}^{\pm}(zq)^{-1}(l_{22}^{\pm}(zq) - l_{21}^{\pm}(zq) l_{11}^{\pm}(zq)^{-1} l_{12}^{\pm}(zq))) \\
&= \triangle(l_{11}^{\pm}(zq)^{-1}(l_{22}^{\pm}(zq) - l_{21}^{\pm}(zq) E^{\pm}(z_{\pm}))) \\
&= [\mathbf{1}\otimes\mathbf{1} + l_{11}^{\pm}(zq^{1\pm c_2/2})^{-1} l_{21}^{\pm}(zq^{1\pm c_2/2}) \otimes l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} l_{12}^{\pm}(zq^{1\mp c_1/2})] \\
&\quad \times [l_{11}^{\pm}(zq^{1\pm c_2/2})^{-1} k_2^{\pm}(zq^{1\pm c_2/2}) \otimes l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} k_2^{\pm}(zq^{1\mp c_1/2}) \\
&\quad - l_{11}^{\pm}(zq^{1\pm c_2/2})^{-1} l_{21}^{\pm}(zq^{1\pm c_2/2}) l_{11}^{\pm}(zq^{1\pm c_2/2})^{-1} k_2^{\pm}(zq^{1\pm c_2/2}) \\
&\quad \otimes l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} k_2^{\pm}(zq^{1\mp c_1/2}) l_{11}^{\pm}(zq^{1\mp c_1/2})^{-1} l_{12}^{\pm}(zq^{1\mp c_1/2})] \\
&= K^{\pm}(zq^{\pm c_2/2}) \otimes K^{\pm}(zq^{\mp c_1/2}) \qquad (44)
\end{aligned}
$$

$$
\begin{aligned}
\triangle(H^{\pm}(z)) &= \triangle(k_1^{\pm}(zq^{-1}) k_2^{\pm}(zq)) = \triangle(k_1^{\pm}(zq^{-1}) k_1^{\pm}(zq) K^{\pm}(z)) \\
&= [l_{11}^{\pm}(zq^{-1\pm c_2/2}) \otimes l_{11}^{\pm}(zq^{-1\mp c_1/2}) - l_{21}^{\pm}(zq^{-1\pm c_2/2}) \otimes l_{12}^{\pm}(zq^{-1\mp c_1/2})] \\
&\quad \times [l_{11}^{\pm}(zq^{1\pm c_2/2}) \otimes l_{11}^{\pm}(zq^{-1\mp c_1/2}) - l_{21}^{\pm}(zq^{1\pm c_2/2}) \otimes l_{12}^{\pm}(zq^{-1\mp c_1/2})] \\
&\quad \times [K^{\pm}(zq^{\pm c_2/2}) \otimes K^{\pm}(zq^{\mp c_1/2})] \\
&= H^{\pm}(zq^{\pm c_2/2}) \otimes H^{\pm}(zq^{\mp c_1/2}) - l_{21}^{\pm}(zq^{-1\pm c_2/2}) l_{11}^{\pm}(zq^{1\pm c_2/2}) K^{\pm}(zq^{\pm c_2/2}) \\
&\quad \otimes K^{\pm}(zq^{\mp c_1/2})(q^{-1} l_{11}^{\pm}(zq^{-1\mp c_1/2}) l_{12}^{\pm}(zq^{1\mp c_1/2}) + l_{12}^{\pm}(zq^{-1\mp c_1/2}) l_{11}^{\pm}(zq^{1\mp c_1/2})) \\
&= H^{\pm}(zq^{\pm c_2/2}) \otimes H^{\pm}(zq^{\mp c_1/2}) - (q+q^{-1}) F^{\pm}(zq^{-2\mp c_1/2\pm c_2/2}) H^{\pm}(zq^{\pm c_2/2}) \\
&\quad \otimes H^{\pm}(zq^{\mp c_1/2}) E^{\pm}(zq^{-2\mp c_1/2\pm c_2/2}). \qquad (45)
\end{aligned}
$$

The antipole and co-unit structure for these currents is

$$S(K^{\pm}(z)) = K^{\pm}(z)^{-1} \qquad (46)$$

$$S(E^{\pm}(z)) = -K^{\pm}(zq^{\pm c/2})^{-1} E^{\pm}(zq^{\pm c}) \qquad (47)$$

$$S(F^{\pm}(z)) = -F^{\pm}(zq^{\mp c}) K^{\pm}(zq^{\mp c/2})^{-1} \qquad (48)$$

$$S(H^{\pm}(z)) = H^{\pm}(z)^{-1} - (q+q^{-1}) F^{\pm}(zq^{2\mp c/2}) H^{\pm}(z)^{-1} K^{\pm}(zq^2)^{-1} E^{\pm}(zq^{2\pm c/2}) \qquad (49)$$

$$\epsilon(K^{\pm}(z)) = \epsilon(H^{\pm}(z)) = \mathbf{1} \qquad (50)$$

$$\epsilon(E^{\pm}(z)) = \epsilon(F^{\pm}(z)) = 0. \qquad (51)$$

The compatibility condition (11) of the antipole and co-product for $E^{\pm}(z)$, $F^{\pm}(z)$, $K^{\pm}(z)$ is easily checked. For simplicity, we do check $m(S\otimes\mathrm{id})\triangle(H^{\pm}(z)) = m(\mathrm{id}\otimes S)\triangle(H^{\pm}(z)) = \mathbf{1}$, using the following relations obtained from (4) and (5)

$$
\begin{aligned}
&E^{\pm}(z_{\pm}q^2) H^{\pm}(z) = q^2 H^{\pm}(z) E^{\pm}(z_{\pm}q^{-2}) \\
&F^{\pm}(z_{\mp}q^{-2}) H^{\pm}(z) = q^2 H^{\pm}(z) F^{\pm}(z_{\mp}q^2) \\
&(q+q^{-1})\{E^{\pm}(z_{\pm}q^2), F^{\pm}(z_{\mp}q^{-2})\} = K^{\pm}(zq^2) - K^{\pm}(zq^{-2})
\end{aligned}
$$

then

$$
\begin{aligned}
m(S\otimes \mathrm{id})\triangle(H^{\pm}(z)) &= S(H^{\pm}(zq^{\pm c/2}))(H^{\pm}(zq^{\pm c/2})\\
&\quad +(q+q^{-1})F^{\pm}(zq^{-2})K^{\pm}(zq^{-2\pm c/2})^{-1}H^{\pm}(zq^{\pm c/2})E^{\pm}(zq^{-2\pm c}))\\
&= \mathbf{1}+(q+q^{-1})H^{\pm}(zq^{\pm c/2})^{-1}F^{\pm}(zq^{-2})\\
&\quad \times H^{\pm}(zq^{\pm c/2})K^{\pm}(zq^{-2\pm c/2})^{-1}E^{\pm}(zq^{-2\pm c})\\
&\quad -(q+q^{-1})F^{\pm}(zq^{2})K^{\pm}(zq^{2\pm c/2})^{-1}\\
&\quad \times H^{\pm}(zq^{\pm c/2})^{-1}E^{\pm}(zq^{2\pm c})H^{\pm}(zq^{\pm c/2}) - (q+q^{-1})^2F^{\pm}(zq^{2})H^{\pm}(zq^{\pm c/2})^{-1}\\
&\quad \times K^{\pm}(zq^{2\pm c/2})^{-1}E^{\pm}(zq^{2\pm c})F^{\pm}(zq^{-2})\\
&\quad \times K^{\pm}(zq^{-2\pm c/2})^{-1}H^{\pm}(zq^{\pm c/2})E^{\pm}(zq^{-2\pm c})\\
&= \mathbf{1}+(q+q^{-1})q^2F^{\pm}(zq^{2})K^{\pm}(zq^{-2\pm c/2})^{-1}E^{\pm}(zq^{-2\pm c})\\
&\quad -(q+q^{-1})q^2F^{\pm}(zq^{2})K^{\pm}(zq^{2\pm c/2})^{-1}E^{\pm}(zq^{-2\pm c})\\
&\quad -(q+q^{-1})q^2F^{\pm}(zq^{2})K^{\pm}(zq^{2\pm c/2})^{-1}(K^{\pm}(zq^{2\pm c/2})\\
&\quad -K^{\pm}(zq^{-2\pm c/2}))K^{\pm}(zq^{-2\pm c/2})^{-1}E^{\pm}(zq^{-2\pm c})\\
&= \mathbf{1}.
\end{aligned}
$$

Similarly, we can also prove

$$m(\mathrm{id}\otimes S)\triangle(H^{\pm}(z)) = \mathbf{1}.$$

The co-product as well as the antipole obtained in this work is different from that in [17]; the relation between them could be found from the so-called twisting, which will be carried out in a forthcoming paper [18]. An analogous procedure can also be applied in studying super Yangian doubles using a rational solution of the super YBE [16].

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Note added in proof. After the completion of this work, we found that the same result has been independently obtained by Zhang [17].

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